Cooperative Robust Parallel Operation of Multiple Actuators

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Abstract

This paper studies the cooperative robust parallel operation of multiple actuators over an undirected communication graph. The plant is modeled as an uncertain linear system, and the actuators are linear and identical. Based on the internal model principle, a distributed dynamic output feedback control law is proposed to achieve both robust output regulation of the closed-loop system and plant input sharing among the actuators. A practical example of five motors cooperatively driving an uncertain shaft under an external load torque is presented to show the effectiveness of the proposed control law.

Problem formulation

Consider the uncertain linear plant (1) and the exosystem (2) again. Different from the plant (1) is driven by a single linear actuator in (3), we consider the scenario where the plant (1) is driven by N identical linear actuators described as follows:

$$\dot{x}_i = A_a x_i + B_a u_i \tag{5a}$$

$$y_i = C_a x_i, \quad i = 1, \ldots, N \tag{5b}$$

$$u_p = \sum_{i=1}^{N} y_i \tag{5c}$$

Then, the cooperative robust parallel operation problem of multiple actuators is achieved if we can find a distributed control law, such that the closed-loop system satisfies: (i). The nominal closed-loop system matrix is Hurwitz; (ii). There exists an open neighborhood W of $w = 0$ such that, for any initial conditions and for all $w \in W$, the solution of the closed-loop system satisfies $\lim_{t\to\infty} e(t) = 0$; (iii). $\lim_{t\to\infty}(y_i(t) - y_j(t)) = 0, i \neq j, i, j = 1, \ldots, N$.

Conclusions

In this paper, we have studied the cooperative robust parallel operation problem where a linear uncertain plant is collectively driven by multiple actuators. Assuming that the communication graph among the actuators is undirected and connected, we have proposed a distributed dynamic output control law based on the internal model principle. Our study demonstrates that, under the same standard assumptions as used in the output regulation literature, both robust output regulation of the closed-loop system and plant input sharing among the actuators can be achieved.

References

[1] J. Huang, *Nonlinear Output Regulation: Theory and Applications*. Philadelphia, PA: SIAM, 2004.

[2] Y. H. Lim and K. K. Oh, "Cooperative parallel operation of multiple actuators," *Automatica*, vol. 162, 111515, 2024.

Linear robust output regulation

Consider an uncertain linear plant described as follows:

$$\dot{x}_p = \bar{A}_p x_p + \bar{B}_p u_p + \bar{E}_p v, \quad e = \bar{C}_p x_p + \bar{F}_p v \tag{1}$$

where $x_p \in \mathbb{R}^{n_p}, u_p \in \mathbb{R}^{m_p}$, and $e \in \mathbb{R}^p$ are the state, the input, and the error output of the plant, respectively; $\bar{A}_p, \bar{B}_p, \bar{C}_p, \bar{E}_p$, and $\bar{F}_p$ are uncertain constant matrices of conformal dimensions; $v \in \mathbb{R}^q$ is the exogenous signal representing the reference input and/or the external disturbance and is assumed to be generated by the following exosystem:

$$\dot{v} = Sv \tag{2}$$

with $S \in \mathbb{R}^{q\times q}$ being a known matrix. Suppose the plant (1) **is driven by an actuator** modeled as

$$\dot{x}_a = A_a x_a + B_a u, \quad u_p = C_a x_a \tag{3}$$

where $x_a \in \mathbb{R}^{n_a}$ and $u \in \mathbb{R}^m$ are the state and the control input of the actuator, respectively, $u_p \in \mathbb{R}^{m_p}$ is both the output of the actuator and the input to the plant, and A_a, B_a, and C_a are constant matrices of conformal dimensions.

Then, **the linear robust output regulation problem** can be solved by the following dynamic output feedback control law:

$$u = Kz, \quad \dot{z} = \mathcal{G}_1 z + \mathcal{G}_2 e \tag{4}$$

Main results

Inspired by [1] and [2], we consider a distributed dynamic output feedback control law of the following form:

$$u_i = Kz_i, \quad \dot{z}_i = \mathcal{G}_1 z_i + \mathcal{G}_2 e + J\sum_{j=1}^{N} a_{ij}(z_j - z_i) \tag{6a}$$

where

- $K =$;
- $\mathcal{G}_1 =$;
- $\mathcal{G}_2 =$;
- $J =$;

A numerical example

In this section, we give a numerical example to demonstrate the effectiveness of our design. The example is adopted from [2] which describes cooperative parallel operation of five electric motors collectively driving a common shaft under an external load torque, see Figure 1. More interestingly, we consider cooperative robust parallel operation that allows the parameters of the plant to be uncertain and undergo perturbations. The simulation results are shown below.